J. Ramanujan Math. Soc. **34,** No.1 (2019) 81–95

Some common fixed point theorems in probabilistic metric space using contractive condition of integral type

U. P. Dolhare[1] and V. V. Nalawade[2]

[1]*D. S. M. College, Jintur, Dist. Parbhani, Maharashtra 431 509, India*
e-mail: uttamdolhare121@gmail.com

[2]*S. G. R. G. Shinde Mahavidyalaya, Paranda, Dist. Osmanabad, Maharashtra 413 502, India*
e-mail: vidyadharnalawade42@gmail.com

Communicated by: Prof. Ritabrata Munshi

Received: July 30, 2017

Abstract. The following notions related to probabilistic metric spaces have been mentioned in the first section of this research article.

(1) Commuting Self Maps,
(2) Weakly Commuting Self Maps,
(3) Compatible Self Maps,
(4) Weakly Compatible Self Maps,
(5) Occasionally Weakly Compatible Self Maps.

While mentioning the above stated concepts, it has been also proved that each pair of self maps satisfies the conditions of its successor, but none of the reverse implication is true. Examples are provided to illustrate these ideas. In the main results, some common fixed point theorems using contractive conditions of integral type in the probabilistic metric spaces are established. An example is presented to validate the results.

2000 Mathematics Subject Classification: 47H10, 54H25.

S. L. Singh, B. D. Pant [23] extended the definition of weakly commuting pair of self maps to probabilistic metric space, S. N. Mishra did the same for compatible pair of self maps [13], B. Singh, S. Jain [22] extended the definition of weak compatibility to probabilistic metric spaces and proved a number of fixed point theorems in this space. H. Chandra, A. Bhatt [3] extended the concept of occasionally weak compatibility to probabilistic metric space. Recently, Chandra and Bhatt [3] proved common fixed point theorems for a pair of occasionally weakly compatible maps in probabilistic semi-metric space. Sastry *et al.* [17] improved the results of Chandra and Bhatt [3]. A good number of interesting and significant results have been obtained by various authors in this direction [2,4,7,10,11,14,16]. Sunny Chauhan *et al.* obtained the fixed point theorems in probabilistic metric space by using contractive condition of integral type [6]. In this paper we establish unique common fixed point theorems for occasionally weakly compatible self maps in probabilistic metric space using a contractive condition of integral type.

2. Preliminary notes

Definition 2.1 ([15]). *A mapping $F : \mathbb{R} \to \mathbb{R}^+$ is said to be a distribution function if it is non-decreasing and left continuous with* $\inf_{t\in\mathbb{R}} F(t) = 0$ *and* $\sup_{t\in\mathbb{R}} F(t) = 1$.

Example 2.2 ([15]). The function defined by

$$H(t) = \begin{cases} 0, & \text{if } t \leq 0 \\ 1, & \text{if } t > 0 \end{cases}$$

is a distribution function.

One more example of distribution function is mentioned ahead. For any two distribution functions F and G, by $F(t) < G(t)$, we mean value of F is less than the value of G at all $t \in \mathbb{R}$. Similarly we can define what we mean by $F(t) > G(t)$ and $F(t) = G(t)$. We shall denote by $\Im$ the set of all distribution functions defined on $[-\infty, \infty]$.

Definition 2.3. *If X is a non-empty set, then $\Gamma : X \times X \to \Im$ is called a probabilistic distance on X. The value of $\Gamma(x, y)$ is usually denoted by $F_{x,y}(t)$, $t \in \mathbb{R}$.*

Definition 2.4 ([15]). *The ordered pair (X, Γ) is called a probabilistic metric space if X is a nonempty set and Γ is a probabilistic distance on X satisfying the following conditions for all $x, y, z \in X$ and $s, t > 0$:*

1) $F_{x,y}(t) = 1$ *for all* $t > 0 \Leftrightarrow x = y$,
2) $F_{x,y}(0) = 0$,
3) $F_{x,y}(t) = F_{y,x}(t)$,
4) *if* $F_{x,y}(t) = 1$ *and* $F_{y,z}(s) = 1$ *then* $F_{x,z}(t + s) = 1$.

If we take $F_{x,y}(t) = H(t - d(x, y))$ in the definition 2.4, we get a usual metric space (X, d) [15]. It is clear that the probabilistic metric spaces are wider spaces than the metric spaces and are better suited for statistical situations.

Definition 2.5. *Let (X, Γ) be a probabilistic metric space. Two self maps f and g of X are said to commute if $F_{fgx,gfx} = 1$ for all $x \in X$. That is self maps f and g of X are said to commute if $fgx = gfx$ for all $x \in X$.*

Example 2.6. Clearly an identity map on X commutes with all the maps on X.

Example 2.7. Consider the functions $f(x) = 2x$ and $g(x) = 3x$. Then we have $fg(x) = f(3x) = 2(3x) = 6x$ and $gf(x) = g(2x) = 3(2x) = 6x$. Therefore $F_{fgx,gfx} = F_{6x,6x} = 1$ and so f and g commute with each other.

Definition 2.8 ([23]). *Let (X, d) be a probabilistic metric space. Then the self maps f and g of X are said to be a weakly commuting pair if $F_{fgx,gfx} \geq F_{fx,gx}$ for $x \in X$. Note that $F_{fgx,gfx} \geq F_{fx,gx}$ imply probabilistic distance between fgx and gfx is greater that or equal to the probabilistic distance between fx and gx.*

Clearly commuting mappings f, g are weakly commuting because then $fg(x) = gf(x)$ and thus $F_{fgx,gfx} = 1 \geq F_{fx,gx}$ for all $x \in X$. But the converse is not true as is revealed by the following example.

Example 2.9. Consider the probabilistic metric space (X, Γ), where $X = [0, \infty)$ and

$$F_{x,y}(t) = \begin{cases} e^{-\frac{|x-y|}{t}}, & \text{if } t > 0 \\ 0, & \text{if } t = 0. \end{cases}$$

Consider the functions $f(x) = \frac{x}{2}$, $g(x) = \frac{x}{2+x}$ defined on X. We observe that

$$\begin{aligned} |fg(x) - gf(x)| &= \left| \frac{x}{4+2x} - \frac{x}{4+x} \right| \\ &= \left| \frac{x^2}{(4+x)(4+2x)} \right| \\ &\leq \left| \frac{x^2}{4+2x} \right| \\ &= \left| \frac{x}{2} - \frac{x}{2+x} \right| \\ &= |fx - gx| \end{aligned}$$

Thus

$$\left| \frac{x}{4+2x} - \frac{x}{4+x} \right| \leq \left| \frac{x}{2} - \frac{x}{2+x} \right|$$

or

$$-\left| \frac{x}{4+2x} - \frac{x}{4+x} \right| \geq -\left| \frac{x}{2} - \frac{x}{2+x} \right|$$

Therefore we get,

$$\begin{aligned} F_{fgx,gfx} &= \exp\left(-\frac{\left|\frac{x}{4+2x} - \frac{x}{4+x}\right|}{t} \right) \\ &\geq \exp\left(-\frac{\left|\frac{x}{2} - \frac{x}{2+x}\right|}{t} \right) \\ &= F_{fx,gx} \end{aligned}$$

That is the functions f, g are weakly commuting. However $fg(x) = f\left(\frac{x}{2+x}\right) = \frac{\left(\frac{x}{2+x}\right)}{2} = \frac{x}{2(2+x)} = \frac{x}{4+2x}$ and $gf(x) = g\left(\frac{x}{2}\right) = \frac{\left(\frac{x}{2}\right)}{2+\left(\frac{x}{2}\right)} = \frac{x}{4+x}$. Thus $fg(x) \neq gf(x)$ and therefore the functions f, g are not commuting.

Gerald Jungck introduced the notion of compatible mappings over the weakly commuting mappings in metric space. S. N. Mishra[13] extended the same to probabilistic metric space as follows.

Definition 2.10 ([13]). *Self mappings f and g of a probabilistic metric space (X, Γ) are compatible if and only if* $\lim_{n\to\infty} F_{fgx_n,gfx_n} = 1$ *whenever $\{x_n\}_{n=1}^{\infty}$ is a sequence in X such that* $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ *for some* $x \in X$.

Remark 2.11. Any weakly commuting pair of self maps is compatible. We observe this as follows. Suppose f, g are weakly commuting mappings on a probabilistic metric space (X, Γ). Then we have $F_{fgx,gfx} \geq F_{fx,gx}$. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence in X such that $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n) = x$ for some $x \in X$. Consider $\lim_{n\to\infty} F_{fgx_n,gfx_n} \geq \lim_{n\to\infty} F_{fx_n,gx_n} = F_{x,x} = 1$. But as $\sup_{n\to\infty} F_{fgx_n,gfx_n} = 1$ we must have $\lim_{n\to\infty} F_{fgx_n,gfx_n} \leq 1$. This imply $\lim_{n\to\infty} F_{fgx_n,gfx_n} = 1$. Thus the pair f, g is compatible.

But the converse may not be true. That is the pair of compatible mappings on a probabilistic metric space may or may not be weakly commuting. This is illustrated in the following example.

Example 2.12. Consider the probabilistic metric space (X, Γ), where $X = [1, \infty)$ and

$$F_{x,y}(t) = \begin{cases} e^{-\frac{|x-y|}{t}}, & \text{if } t > 0 \\ 0, & \text{if } t = 0 \end{cases}$$

Consider the functions $f(x) = \cosh(x)$ and $g(x) = \sinh(x)$ defined on X. Then f and g are compatible. (In this case the condition of compatibility is vacuously satisfied because there is no sequence $\{x_n\}_{n=1}^{\infty}$ for which the sequences $\{\cosh(x_n)\}_{n=1}^{\infty}$ and $\{\sinh(x_n)\}_{n=1}^{\infty}$ converge to an element x of X). Now we see that,

$$|\cosh(\sinh(x)) - \sinh(\cosh(x))| > |\cosh(x) - \sinh(x)|$$
$$\Rightarrow -|\cosh(\sinh(x)) - \sinh(\cosh(x))| < -|\cosh(x) - \sinh(x)|$$

We can see that $|\cosh(\sinh(x)) - \sinh(\cosh(x))| > |\cosh(x) - \sinh(x)|$ from the following figure.

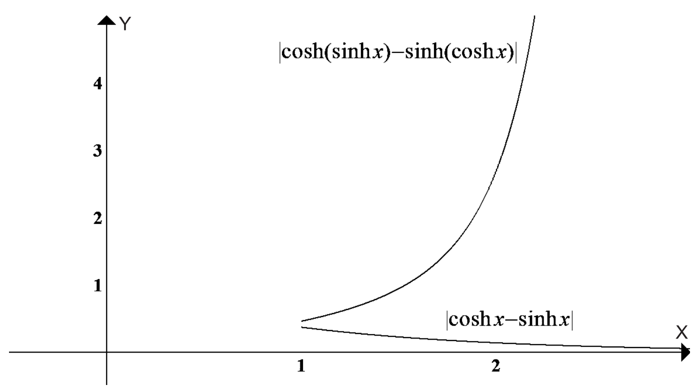

Figure 1. Graph showing $|\cosh(\sinh(x)) - \sinh(\cosh(x))| > |\cosh(x) - \sinh(x)|$ on $X = [1, \infty)$

So we get

$$\begin{aligned} F_{fgx,gfx} &= \exp\left(-\frac{|\cosh(\sinh(x)) - \sinh(\cosh(x))|}{t}\right) \\ &< \exp\left(-\frac{|\cosh(x) - \sinh(x)|}{t}\right) \\ &= F_{fx,gx} \end{aligned}$$

Thus the functions f and g are not weakly commuting.

Definition 2.13 ([22]). *Let (X, Γ) be a probabilistic metric space. Then $x \in X$ is called a coincidence point of two self maps f and g if $fx = gx$. We call $w = fx = gx$ a point of coincidence of f and g.*

Definition 2.14 ([22]). *Two self maps f and g of a probabilistic metric space (X, Γ) are said to be weakly compatible if $F_{fgx,gfx} = 1$ whenever $fx = gx$ for $x \in X$.*

Remark 2.15. Every pair of compatible maps on probabilistic metric space is weakly compatible. Indeed, suppose f and g are compatible self maps of a probabilistic metric space (X, Γ). Suppose $x \in X$ be a coincidence point of f and g, that is $fx = gx$. Consider the sequence $\{x_n\}_{n=1}^{\infty} = \{x\}_{n=1}^{\infty} = \{x, x, \ldots\}$. Then clearly $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} f(x) = f(x)$ and $\lim_{n\to\infty} g(x_n) = \lim_{n\to\infty} g(x) = g(x)$. But as $f(x) = g(x) \in X$, we have $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n)$ and the limit exists in X. So by compatibility of f and g, we have $\lim_{n\to\infty} F_{fgx_n,gfx_n} = F_{fgx,gfx} = 1$. Thus f, g are weakly compatible. But the converse may not be true as is shown in the example below.

Example 2.16. Consider the probabilistic metric space (X, Γ), where $X = [0, 20]$ and

$$F_{x,y}(t) = \begin{cases} e^{-\frac{|x-y|}{t}}, & \text{if } t > 0 \\ 0, & \text{if } t = 0 \end{cases}$$

Define f, g on X as follows:

$$f(x) = \begin{cases} 0, & \text{if } x = 0 \\ x + 7, & \text{if } 0 < x \leq 7 \\ x - 7, & \text{if } 7 < x \leq 20 \end{cases}$$

$$g(x) = \begin{cases} 0, & \text{if } x = 0 \\ 5, & \text{if } 0 < x \leq 7 \\ 0, & \text{if } 7 < x \leq 20 \end{cases}$$

Let $\{x_n\}_{n=1}^{\infty} = \left\{7 + \frac{1}{n}\right\}_{n=1}^{\infty}$ be the sequence of points in X. Then $f(x_n) = f\left(7 + \frac{1}{n}\right) = \left(7 + \frac{1}{n}\right) - 7 = \frac{1}{n}$ and $g(x_n) = g\left(7 + \frac{1}{n}\right) = 0$. Thus $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n) = 0$. But $\lim_{n\to\infty} F_{fgx_n,gfx_n} = \lim_{n\to\infty} F_{0,5} = e^{-\frac{|0-5|}{t}} = e^{-\frac{5}{t}} \neq 1$ for all $t \in \mathbb{R}$. Thus f, g are not compatible on X. However f, g are weakly compatible because they commute at their coincidence point $x = 0$, because $f(0) = g(0) = 0$, $fg(0) = gf(0) = 0$.

Definition 2.17. *Let (X, Γ) be a probabilistic metric space and f, g are self maps of X. Then f, g are said to be occasionally weakly compatible (owc) if there exists at least one point $x \in X$ which is a coincidence point of f and g and at which they commute, that is, there exists $x \in X$ such that $fx = gx$, $fgx = gfx$.*

Remark 2.18. Clearly every weakly compatible pair of maps is owc. The following example shows that the owc pair of self maps may not be always weakly compatible. Thus the notion of occasionally weakly compatible mappings is more general than the notion of weakly compatible mappings.

Example 2.19. Let (X, Γ) be a probabilistic metric space, where $X = \mathbb{R}$ and

$$F_{x,y} = \begin{cases} e^{-\frac{|x-y|}{t}}, & \text{if } t > 0 \\ 0, & \text{if } t = 0 \end{cases}$$

Define $f, g : X \to X$ by $f(x) = 5x$ and $g(x) = x^2$ for all $x \in X$. Then $f(x) = g(x)$ for $x = 0, 5$. But $fg(0) = gf(0) = 0$ and $fg(5) = 125 \neq 625 = gf(5)$. Thus f, g are owc maps but not weakly compatible.

There are many fixed point results related to the probabilistic metric spaces. We refer to some of them as follows.

Lemma 2.20 ([5]). *Let (X, Γ) be a probabilistic metric space and f, g are occasionally weakly compatible self maps of X. If f, g have a unique point of coincidence $w = fx = gx$, then w is the unique fixed point of f and g.*

The following result is obtained by Sunny Chauhan *et al.*

Theorem 2.21 ([5]). *Let (X,Γ) be a probabilistic metric space. Further let A, B, S and T be self maps of X and the pairs (A,S) and (B,T) be each owc maps satisfying $F_{Ax,By}(\phi(t)) \geq \min\{F_{Sx,Ty}(t), F_{Ax,Sx}(t), F_{By,Ty}(t)\}$ for all $x, y \in X$ and $t > 0$. Here the function $\phi(t) : [0,\infty) \to [0,\infty)$ is onto, strictly increasing and satisfies $\sum_{n=1}^{\infty} \phi^{(n)}(t) < \infty$ for all $t > 0$, where $\phi^{(n)}(t)$ denotes the n^{th} derivative of $\phi(t)$. Then there is unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of A, B, S and T.*

Taking $A = B$ and $S = T$ in the above theorem 2.21 we get the following result due to Sunny Chauhan *et al.*

Theorem 2.22 ([5]). *Let (X,Γ) be a probabilistic metric space. Further let A and S be self maps of X and the pair (A,S) be owc satisfying $F_{Ax,Ay}(\phi(t)) \geq \min\{F_{Sx,Sy}(t), F_{Ax,Sx}(t), F_{Ay,Sy}(t)\}$ for all $x, y \in X$ and $t > 0$. Here the function $\phi(t) : [0,\infty) \to [0,\infty)$ is onto, strictly increasing and satisfies $\sum_{n=1}^{\infty} \phi^{(n)}(t) < \infty$ for all $t > 0$, where $\phi^{(n)}(t)$ denotes the n^{th} derivative of $\phi(t)$. Then there is unique common fixed point of A and S.*

Further taking $A = B$ in the above theorem 2.21, we get the following interesting result.

Theorem 2.23 ([5]). *Let (X,Γ) be a probabilistic metric space. Further let A, S and T be self maps of X and the pairs (A,S) and (A,T) be each owc maps satisfying $F_{Ax,Ay}(\phi(t)) \geq \min\{F_{Sx,Ty}(t), F_{Ax,Sx}(t), F_{Ay,Ty}(t)\}$ for all $x, y \in X$ and $t > 0$. Here the function $\phi(t) : [0,\infty) \to [0,\infty)$ is onto, strictly increasing and satisfies $\sum_{n=1}^{\infty} \phi^{(n)}(t) < \infty$ for all $t > 0$, where $\phi^{(n)}(t)$ denotes the n^{th} derivative of $\phi(t)$. Then there is a unique common fixed point of A, S and T in X.*

The following results are due to Pant, B. D. *et al.*

Theorem 2.24 ([15]). *Let (X,Γ) be a probabilistic metric space. Further let (L,A) and (M,S) are occasionally weakly compatible maps in X satisfying*

$$\min\{F_{Lx,My}(kt), F_{Sy,Lx}(kt)\} + \gamma F_{Sy,My}(kt) \geq \alpha F_{Ax,Lx}(t) + \beta F_{Ax,Sy}(t)$$

for all $x, y \in X$, $k \in (0,1)$, $t > 0$, where $0 < \alpha, \beta < 1$, $0 \leq \gamma < 1$ such that $\alpha + \beta - \gamma = 1$. Then L, A, M and S have a unique common fixed point in X.

Theorem 2.25 ([15]). *Let (X,Γ) be a probabilistic metric space. Further let (L,A) be occasionally weakly compatible pair of maps in X satisfying*

$$\min\{F_{Lx,Ly}(kt), F_{Ay,Lx}(kt)\} + \gamma F_{Ay,Ly}(kt) \geq \alpha F_{Ax,Lx}(t) + \beta F_{Ax,Ay}(t)$$

for all $x, y \in X$, $k \in (0,1)$, $t > 0$, where $o < \alpha, \beta < 1$, $0 \leq \gamma < 1$ such that $\alpha + \beta - \gamma = 1$. Then L and A have a unique common fixed point in X.

3. Main results

We extend the theorems 2.21–2.25 as follows.

Theorem 3.1. *Let (X,Γ) be a probabilistic metric space. Let f, g, S and T be self maps of X and the pairs (f,S) and (g,T) are each occasionally weakly compatible. If*

$$\int_0^{F_{fx,gy}} \phi(s)ds > \int_0^{m(x,y)} \phi(s)ds \tag{1}$$

for each $x, y \in X$, where $m(x,y) = \min\{F_{Sx,Ty}, F_{Sx,fx}, F_{Ty,gy}, F_{Sx,gy}, F_{Ty,fx}\}$ and $\phi : [0,\infty) \to [0,\infty)$ is a Lebesgue integrable mapping which is mapping on each compact subset of $[0,\infty)$ and such that for all $\epsilon > 0$,

$$\int_0^{\epsilon} \phi(s)ds > 0. \tag{2}$$

Then there is a unique point $w \in X$ such that $fw = Sw = w$ and a unique point $z \in X$ such that $gz = Tz = z$. Moreover, $z = w$, so that there is unique common fixed point of f, g, S and T.

Proof. Since the pairs (f, S) and (g, T) are each owc, there exists points $x, y \in X$ such that $fx = Sx$, $fSx = Sfx$ and $gy = Ty$, $gTy = Tgy$. We prove $fx = gy$. Suppose $fx \neq gy$. Therefore we obtain

$$\begin{aligned} m(x, y) &= \min\{F_{Sx,Ty}, F_{Sx,fx}, F_{Ty,gy}, F_{Sx,gy}, F_{Ty,fx}\} \\ &= \min\{F_{fx,gy}, F_{fx,fx}, F_{gy,gy}, F_{fx,gy}, F_{gy,fx}\} \\ &= \min\{F_{fx,gy}, 1\} \\ &= F_{fx,gy}\left(\because \sup_{fx,gy\in X} F_{fx,gy} = 1 \Rightarrow F_{fx,gy} \leq 1\right) \end{aligned} \tag{3}$$

From inequality (1) we get,

$$\int_0^{F_{fx,gy}} \phi(s)ds > \int_0^{m(x,y)} \phi(s)ds = \int_0^{F_{fx,gy}} \phi(s)ds \tag{4}$$

This is a contradiction. Therefore $fx = gy$. Thus $fx = Sx = gy = Ty$. Moreover, if there is another point z such that $fz = Sz$ and $fz \neq gy$, then again inequality (1) imply a contradiction that

$$\int_0^{F_{fz,gy}} \phi(s)ds > \int_0^{m(z,y)} \phi(s)ds = \int_0^{F_{fz,gy}} \phi(s)ds. \tag{5}$$

Therefore we have $fz = gy$. It follows that $fz = Sz = gy = Ty$ or $fx = fz$ and $Sx = Sz$. So that there is unique $x \in X$ such that $fx = Sx$. Let $w = fx = Sx$ be the unique point of coincidence of f and S. By lemma 2.20 w is unique common fixed point of f and S. Also there is unique point $z \in X$ such that $z = gz = Tz$. Suppose, $w \neq z$. Then

$$\begin{aligned} m(w, z) &= \min\{F_{Sw,Tz}, F_{Sw,fw}, F_{Tz,gz}, F_{Sw,gz}, F_{Tz,fw}\} \\ &= \min\{F_{w,z}, F_{w,w}, F_{z,z}, F_{w,z}, F_{z,w}\} \\ &= \min\{F_{w,z}, 1\} \\ &= F_{w,z}\left(\because \sup_{w,z\in X} F_{w,z} = 1\right) \end{aligned} \tag{6}$$

Using inequality (1), we get

$$\int_0^{F_{w,z}} \phi(s)ds > \int_0^{m(w,z)} \phi(s)ds = \int_0^{F_{w,z}} \phi(s)ds. \tag{7}$$

This is a contradiction. Therefore $w = z$ and w is a unique common fixed point of f, g, S and T. □

Theorem 3.2. *Let (X, Γ) be a probabilistic metric space. Let f, g, S and T be self maps of X and the pairs (f, S) and (g, T) are each occasionally weakly compatible. If*

$$\int_0^{F_{fx,gy}} \phi(s)ds > h\int_0^{m(x,y)} \phi(s)ds. \tag{8}$$

for each $x, y \in X$, where $h > 1$, $m(x, y) = \min\left\{F_{Sx,Ty}, F_{Sx,fx}, F_{Ty,gy}, \frac{F_{Sx,gy}+F_{Ty,fx}}{2}\right\}$ and $\phi : [0,\infty) \to [0,\infty)$ is a Lebesgue integrable mapping which is mapping on each compact subset of $[0,\infty)$ and such that for all $\epsilon > 0$,

$$\int_0^{\epsilon} \phi(s)ds > 0. \tag{9}$$

Then there is a unique point $w \in X$ such that $fw = Sw = w$ and a unique point $z \in X$ such that $gz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of f, g, S and T.

Proof. Since the pairs (f, S) and (g, T) are each owc, there exists points $x, y \in X$ such that $fx = Sx$, $fSx = Sfx$ and $gy = Ty$, $gTy = Tgy$. We prove $fx = gy$. Suppose $fx \neq gy$. Therefore we obtain

$$\begin{aligned} m(x, y) &= \min\left\{F_{Sx,Ty}, F_{Sx,fx}, F_{Ty,gy}, \frac{F_{Sx,gy} + F_{Ty,fx}}{2}\right\} \\ &= \min\left\{F_{fx,gy}, F_{fx,fx}, F_{gy,gy}, \frac{F_{fx,gy} + F_{gy,fx}}{2}\right\} \\ &= \min\{F_{fx,gy}, 1\} \\ &= F_{fx,gy}\left(\because \sup_{fx,gy\in X} F_{fx,gy} = 1\right) \end{aligned} \tag{10}$$

From inequality (8) we get,

$$\int_0^{F_{fx,gy}} \phi(s)ds > h\int_0^{m(x,y)} \phi(s)ds = h\int_0^{F_{fx,gy}} \phi(s)ds. \tag{11}$$

This is a contradiction. Therefore $fx = gy$. Thus $fx = Sx = gy = Ty$. Moreover, if there is another point z such that $fz = Sz$ and $fz \neq gy$, then again inequality (8) imply a contradiction that

$$\int_0^{F_{fz,gy}} \phi(s)ds > h\int_0^{m(z,y)} \phi(s)ds = h\int_0^{F_{fz,gy}} \phi(s)ds. \tag{12}$$

Therefore we have $fz = gy$. It follows that $fz = Sz = gy = Ty$ or $fx = fz$ and $Sx = Sz$. So that there is unique $x \in X$ such that $fx = Sx$. Let $w = fx = Sx$ be the unique point of coincidence of f and S. By lemma 2.20 w is unique common fixed point of f and S. Also there is unique point $z \in X$ such that $z = gz = Tz$. Suppose, $w \neq z$. Then

$$\begin{aligned} m(w, z) &= \min\left\{F_{Sw,Tz}, F_{Sw,fw}, F_{Tz,gz}, \frac{F_{Sw,gz} + F_{Tz,fw}}{2}\right\} \\ &= \min\left\{F_{w,z}, F_{w,w}, F_{z,z}, \frac{F_{w,z} + F_{z,w}}{2}\right\} \\ &= \min\{F_{w,z}, 1\} \\ &= F_{w,z}\left(\because \sup_{w,z\in X} F_{w,z} = 1\right). \end{aligned} \tag{13}$$

Using inequality (8), we get

$$\int_0^{F_{w,z}} \phi(s)ds > h\int_0^{m(w,z)} \phi(s)ds = h\int_0^{F_{w,z}} \phi(s)ds. \tag{14}$$

This is a contradiction. Therefore $w = z$ and w is a unique common fixed point of f, g, S and T. □

Definition 3.3. *Let (X, Γ) be a probabilistic metric space. A Symmetric on X is a mapping $R \in \Im$ such that*

1) $R_{x,y}(t) = 1$ *for all* $t > 0 \Leftrightarrow x = y$,
2) $R_{x,y}(t) = R_{y,x}(t)$ *for all* $x, y \in X$.

Theorem 3.4. *Let (X, Γ) be a probabilistic metric space with symmetric R. Let f, g, S and T be self maps of X and the pairs (f, S) and (g, T) are each occasionally weakly compatible. If*

$$\int_0^{R_{fx,gy}} \phi(s)ds > \int_0^{m(x,y)} \phi(s)ds \tag{15}$$

for each $x, y \in X$ *such that* $fx \neq gy$ *where,*

$$m(x, y) = \min\{R_{Sx,Ty}, R_{Sx,fx}, R_{Ty,gy}, R_{Sx,gy}, R_{Ty,fx}\}$$

and $\phi : [0, \infty) \to [0, \infty)$ *is a Lebesgue integrable mapping which is mapping on each compact subset of* $[0, \infty)$ *and such that for all* $\epsilon > 0$,

$$\int_0^{\epsilon} \phi(s)ds > 0. \tag{16}$$

Then there is a unique point $w \in X$ *such that* $fw = Sw = w$ *and a unique point* $z \in X$ *such that* $gz = Tz = z$. *Moreover,* $z = w$, *so that there is a unique common fixed point of* f, g, S *and* T.

Proof. Since the pairs (f, S) and (g, T) are each owc, there exists points $x, y \in X$ such that $fx = Sx$, $fSx = Sfx$ and $gy = Ty$, $gTy = Tgy$. We prove $fx = gy$. Suppose $fx \neq gy$. Therefore we obtain

$$\begin{aligned} m(x, y) &= \min\{R_{Sx,Ty}, R_{Sx,fx}, R_{Ty,gy}, R_{Sx,gy}, R_{Ty,fx}\} \\ &= \min\{R_{fx,gy}, R_{fx,fx}, R_{gy,gy}, R_{fx,gy}, R_{gy,fx}\} \\ &= \min\{R_{fx,gy}, 1\} \\ &= R_{fx,gy}\left(\because \sup_{fx,gy\in X} R_{fx,gy} = 1 \Rightarrow R_{fx,gy} \leq 1\right) \end{aligned} \tag{17}$$

From inequality (15) we get,

$$\int_0^{R_{fx,gy}} \phi(s)ds > \int_0^{m(x,y)} \phi(s)ds = \int_0^{R_{fx,gy}} \phi(s)ds \tag{18}$$

This is a contradiction. Therefore $fx = gy$. Thus $fx = Sx = gy = Ty$. Moreover, if there is another point z such that $fz = Sz$ and $fz \neq gy$, then again inequality (15) imply a contradiction that

$$\int_0^{R_{fz,gy}} \phi(s)ds > \int_0^{m(z,y)} \phi(s)ds = \int_0^{R_{fz,gy}} \phi(s)ds. \tag{19}$$

Therefore we have $fz = gy$. It follows that $fz = Sz = gy = Ty$ or $fx = fz$ and $Sx = Sz$. So that there is unique $x \in X$ such that $fx = Sx$. Let $w = fx = Sx$ be the unique point of coincidence of f and S. By lemma 2.20 w is unique common fixed point of f and S. Also there is unique point $z \in X$ such that $z = gz = Tz$. Suppose, $w \neq z$. Then

$$\begin{aligned} m(w, z) &= \min\{R_{Sw,Tz}, R_{Sw,fw}, R_{Tz,gz}, R_{Sw,gz}, R_{Tz,fw}\} \\ &= \min\{R_{w,z}, R_{w,w}, R_{z,z}, R_{w,z}, R_{z,w}\} \\ &= \min\{R_{w,z}, 1\} \\ &= R_{w,z}\left(\because \sup_{w,z\in X} R_{w,z} = 1\right) \end{aligned} \tag{20}$$

Using inequality (15), we get

$$\int_0^{R_{w,z}} \phi(s)ds > \int_0^{m(w,z)} \phi(s)ds = \int_0^{R_{w,z}} \phi(s)ds. \tag{21}$$

This is a contradiction. Therefore $w = z$ and w is a unique common fixed point of f, g, S and T. □

Theorem 3.5. *Let (X, Γ) be a probabilistic metric space. Let f, g, S and T be self maps of X and the pairs (f, S) and (g, T) are each occasionally weakly compatible. If*

$$\int_0^{R_{fx,gy}} \phi(s)ds > h \int_0^{m(x,y)} \phi(s)ds \tag{22}$$

for each $x, y \in X$, where $h > 1$, $fx \neq gy$,

$$m(x, y) = \min\left\{R_{Sx,Ty}, R_{Sx,fx}, R_{Ty,gy}, \frac{R_{Sx,gy} + R_{Ty,fx}}{2}\right\}$$

and $\phi : [0, \infty) \to [0, \infty)$ is a Lebesgue integrable mapping which is mapping on each compact subset of $[0, \infty)$ and such that for all $\epsilon > 0$,

$$\int_0^{\epsilon} \phi(s)ds > 0. \tag{23}$$

Then there is a unique point $w \in X$ such that $fw = Sw = w$ and a unique point $z \in X$ such that $gz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of f, g, S and T.

Proof. Since the pairs (f, S) and (g, T) are each owc, there exists points $x, y \in X$ such that $fx = Sx$, $fSx = Sfx$ and $gy = Ty$, $gTy = Tgy$. We prove $fx = gy$. Suppose $fx \neq gy$. Therefore we obtain

$$\begin{aligned} m(x, y) &= \min\left\{R_{Sx,Ty}, R_{Sx,fx}, R_{Ty,gy}, \frac{R_{Sx,gy} + R_{Ty,fx}}{2}\right\} \\ &= \min\left\{R_{fx,gy}, R_{fx,fx}, R_{gy,gy}, \frac{R_{fx,gy} + R_{gy,fx}}{2}\right\} \\ &= \min\{R_{fx,gy}, 1\} \\ &= R_{fx,gy}\left(\because \sup_{fx,gy\in X} R_{fx,gy} = 1\right) \end{aligned} \tag{24}$$

From inequality (22) we get,

$$\int_0^{R_{fx,gy}} \phi(s)ds > h \int_0^{m(x,y)} \phi(s)ds = h \int_0^{R_{fx,gy}} \phi(s)ds. \tag{25}$$

This is a contradiction. Therefore $fx = gy$. Thus $fx = Sx = gy = Ty$. Moreover, if there is another point z such that $fz = Sz$ and $fz \neq gy$, then again inequality (22) imply a contradiction that

$$\int_0^{R_{fz,gy}} \phi(s)ds > h \int_0^{m(z,y)} \phi(s)ds = h \int_0^{R_{fz,gy}} \phi(s)ds. \tag{26}$$

Therefore we have $fz = gy$. It follows that $fz = Sz = gy = Ty$ or $fx = fz$ and $Sx = Sz$. So that there is unique $x \in X$ such that $fx = Sx$. Let $w = fx = Sx$ be the unique point of coincidence of f and S. By lemma 2.20 w is unique common fixed point of f and S. Also there is unique point $z \in X$ such that $z = gz = Tz$. Suppose, $w \neq z$. Then

$$\begin{aligned} m(w, z) &= \min\left\{R_{Sw,Tz}, R_{Sw,fw}, R_{Tz,gz}, \frac{R_{Sw,gz} + R_{Tz,fw}}{2}\right\} \\ &= \min\left\{R_{w,z}, R_{w,w}, R_{z,z}, \frac{R_{w,z} + R_{z,w}}{2}\right\} \\ &= \min\{R_{w,z}, 1\} \\ &= R_{w,z}\left(\because \sup_{w,z\in X} R_{w,z} = 1\right) \end{aligned} \tag{27}$$

Using inequality (22), we get

$$\int_0^{R_{w,z}} \phi(s)ds > h \int_0^{m(w,z)} \phi(s)ds = h \int_0^{R_{w,z}} \phi(s)ds. \tag{28}$$

This is a contradiction. Therefore $w = z$ and w is a unique common fixed point of f, g, S and T. □

Example 3.6. Let (X, Γ) be a probabilistic metric space, where $X = [2, 20]$ and

$$F_{x,y}(t) = \begin{cases} e^{-\frac{|x-y|}{t}}, & \text{if } t > 0 \\ 0, & \text{if } t = 0 \end{cases}$$

for all $x, y \in X$.

Define f, S, g and T on X as follows.

$$f(x) = \begin{cases} 2, & \text{if } x = 2, \\ 3, & \text{if } 2 < x \leq 20 \end{cases}$$

$$S(x) = \begin{cases} 2, & \text{if } x = 2, \\ 6, & \text{if } 2 < x \leq 20 \end{cases}$$

$$g(x) = \begin{cases} 2, & \text{if } x = 2, \\ 6, & \text{if } 2 < x \leq 5, \\ 2, & \text{if } 5 < x \leq 20 \end{cases}$$

$$T(x) = \begin{cases} 2, & \text{if } x = 2, \\ 12, & \text{if } 2 < x \leq 5, \\ x - 3, & \text{if } 5 < x \leq 20. \end{cases}$$

Let $\phi(t) = t$ for $t > 0$ and $\phi(0) = 0$. If we choose $\{x_n\}_{n=1}^{\infty} = \{5 + \frac{1}{n}\}_{n=1}^{\infty}$, then $\lim_{n\to\infty} T(x_n) = \lim_{n\to\infty} T\big(5 + \frac{1}{n}\big) = \lim_{n\to\infty} \big(5 + \frac{1}{n} - 3\big) = 2$ and $\lim_{n\to\infty} g(x_n = \lim_{n\to\infty} \big(5 + \frac{1}{n}\big) = \lim_{n\to\infty} 2 = 2$. But $Tg(x_n) = Tg\big(5 + \frac{1}{n}\big) = T(2) = 2$ and $gT(x_n) = gT\big(5 + \frac{1}{n}\big) = g\big(5 + \frac{1}{n} - 3\big) = g\big(2 + \frac{1}{n}\big) = 6$. Thus $Tg(x_n) \neq gT(x_n)$. So g and T are not compatible. But $g(2) = 2 = T(2)$. Also $Tg(2) = T(2) = 2$ and $gT(2) = g(2) = 2$. Thus g and T are occasionally weakly compatible. Now we show that inequality (1) is satisfied for all $x, y \in X$. We consider following four cases for x and y.

case 1) $2 < x < 5, 2 < y < 5$
case 2) $2 < x < 5, 5 < y < 20$
case 3) $5 < x < 20, 2 < y < 5$
case 4) $5 < x < 20, 5 < y < 20$

Consider the above four cases one by one.

Case 1. $2 < x < 5, 2 < y < 5$.
In this case

$$\int_0^{F_{fx,gy}} \phi(s)ds = \int_0^{F_{3,6}} sds = \int_0^{e^{-\frac{3}{t}}} sds = \left[\frac{s^2}{2}\right]_0^{e^{-\frac{3}{t}}} = \frac{e^{-\frac{6}{t}}}{2}$$

$$\begin{aligned} m(x,y) &= \min\{F_{Sx,Ty}, F_{Sx,fx}, F_{Ty,gy}, F_{Sx,gy}, F_{Ty,fx}\} \\ &= \min\left\{F_{6,12}, F_{6,3}, F_{12,6}, F_{6,6}, F_{12,3}\right\} \\ &= \min\left\{e^{-\frac{6}{t}}, e^{-\frac{3}{t}}, e^{-\frac{6}{t}}, 1, e^{-\frac{9}{t}}\right\} \\ &= e^{-\frac{9}{t}} \end{aligned}$$

So that we get,

$$\int_0^{m(x,y)} \phi(s)ds = \int_0^{e^{-\frac{9}{t}}} sds = \left(\frac{s^2}{2}\right)_0^{-\frac{9}{t}} = \frac{-\frac{18}{t}}{2}.$$

Thus

$$\int_0^{F_{fx,gy}} \phi(s)ds = \frac{e^{-\frac{6}{t}}}{2} > \frac{e^{-\frac{18}{t}}}{2} = \int_0^{m(x,y)} \phi(s)ds.$$

Case 2. $2 < x < 5, 5 < y < 20.$

In this case

$$\int_0^{F_{fx,gy}} \phi(s)ds = \int_0^{F_{3,2}} sds = \int_0^{e^{-\frac{1}{t}}} sds = \left(\frac{s^2}{2}\right)_0^{e^{-\frac{1}{t}}} = \frac{e^{-\frac{2}{t}}}{2}.$$

$$\begin{aligned} m(x,y) &= \min\{F_{Sx,Ty}, F_{Sx,fx}, F_{Ty,gy}, F_{Sx,gy}, F_{Ty,fx}\} \\ &= \min\{F_{6,y-3}, F_{6,3}, F_{y-3,2}, F_{6,2}, F_{y-3,3}\} \\ &= \min\left\{e^{-\frac{|y-9|}{t}}, e^{-\frac{3}{t}}, e^{-\frac{|y-5|}{t}}, e^{-\frac{4}{t}}, e^{-\frac{|y-6|}{t}}\right\} \\ &= \min\left\{e^{-\frac{11}{t}}, e^{-\frac{3}{t}}, e^{-\frac{15}{t}}, e^{-\frac{4}{t}}, e^{-\frac{14}{t}}\right\} \\ &= e^{-\frac{15}{t}} \end{aligned}$$

So that we get

$$\int_0^{m(x,y)} \phi(s)ds = \int_0^{e^{-\frac{15}{t}}} sds = \left[\frac{s^2}{2}\right]_0^{e^{-\frac{15}{t}}} = \frac{e^{-\frac{30}{t}}}{2}.$$

Thus

$$\int_0^{F_{fx,gy}} \phi(s)ds = \frac{e^{-\frac{2}{t}}}{2} > \frac{e^{-\frac{30}{t}}}{2} = \int_0^{m(x,y)} \phi(s)ds.$$

Case 3. $5 < x < 20, 2 < y < 5$.

In this case

$$\int_0^{F_{fx,gy}} \phi(s)ds = \int_0^{F_{3,6}} sds = \int_0^{e^{-\frac{3}{t}}} sds = \left[\frac{s^2}{2}\right]_0^{-\frac{3}{t}} = \frac{e^{-\frac{6}{t}}}{2}.$$

$$\begin{aligned} m(x, y) &= \min\{F_{Sx,Ty}, F_{Sx,fx}, F_{Ty,gy}, F_{Sx,gy}, F_{Ty,fx}\} \\ &= \min\{F_{6,12}, F_{6,3}, F_{12,6}, F_{6,6}, F_{12,3}\} \\ &= \min\left\{e^{-\frac{6}{t}}, e^{-\frac{3}{t}}, e^{-\frac{6}{t}}, 1, e^{-\frac{9}{t}}\right\} \\ &= e^{-\frac{9}{t}} \end{aligned}$$

So that we get,

$$\int_0^{m(x,y)} \phi(s)ds = \int_0^{e^{-\frac{9}{t}}} sds = \left[\frac{s^2}{2}\right]_0^{e^{-\frac{9}{t}}} = \frac{e^{-\frac{18}{t}}}{2}.$$

Thus

$$\int_0^{F_{fx,gy}} \phi(s)ds = \frac{e^{-\frac{6}{t}}}{2} > \frac{e^{-\frac{18}{t}}}{2} = \int_0^{m(x,y)} \phi(s)ds.$$

Case 4. $5 < x < 20, 5 < y < 20$.

In this case

$$\int_0^{F_{fx,gy}} \phi(s)ds = \int_0^{F_{3,2}} sds = \int_0^{e^{-\frac{1}{t}}} sds = \left[\frac{s^2}{2}\right]_0^{-\frac{1}{t}} = \frac{e^{-\frac{2}{t}}}{2}.$$

$$\begin{aligned} m(x, y) &= \min\{F_{Sx,Ty}, F_{Sx,fx}, F_{Ty,gy}, F_{Sx,gy}, F_{Ty,fx}\} \\ &= \min\{F_{6,y-3}, F_{6,3}, F_{y-3,2}, F_{6,2}, F_{y-3,3}\} \\ &= \min\left\{e^{-\frac{|y-9|}{t}}, e^{-\frac{3}{t}}, e^{-\frac{|y-5|}{t}}, e^{-\frac{4}{t}}, e^{-\frac{|y-6|}{t}}\right\} \\ &= \min\left\{e^{-\frac{11}{t}}, e^{-\frac{3}{t}}, e^{-\frac{15}{t}}, e^{-\frac{4}{t}}, e^{-\frac{14}{t}}\right\} \\ &= e^{-\frac{15}{t}} \end{aligned}$$

So that we get,

$$\int_0^{m(x,y)} \phi(s)ds = \int_0^{e^{-\frac{15}{t}}} sds = \left[\frac{s^2}{2}\right]_0^{e^{-\frac{15}{t}}} = \frac{e^{-\frac{30}{t}}}{2}.$$

Thus

$$\int_0^{F_{fx,gy}} \phi(s)ds = \frac{e^{-\frac{2}{t}}}{2} > \frac{e^{-\frac{30}{t}}}{2} = \int_0^{m(x,y)} \phi(s)ds.$$

Thus we have

$$\int_0^{F_{fx,gy}} \phi(s)ds > \int_0^{m(x,y)} \phi(s)ds$$

for all $x, y \in X$ and the condition (1) is satisfied. We see that $x = 2$ is the unique common fixed point of f, g, S and T.

Taking $f = g$ and $S = T$ in the theorem 3.1 we get the following result.

Corollary 3.7. *Let (X, Γ) be a probabilistic metric space. Let f and S be self maps of X and the pair (f, S) be occasionally weakly compatible. If*

$$\int_0^{F_{fx,fy}} \phi(s)ds > \int_0^{m(x,y)} \phi(s)ds \tag{29}$$

for each $x, y \in X$, where $m(x, y) = \min\{F_{Sx,Sy}, F_{Sx,fx}, F_{Sy,fy}, F_{Sx,fy}, F_{Sy,fx}\}$ and $\phi : [0, \infty) \to [0, \infty)$ is a Lebesgue integrable mapping which mapping on each compact subset of $[0, \infty)$ and such that for all $\epsilon > 0$,

$$\int_0^{\epsilon} \phi(s)ds > 0. \tag{30}$$

Then there is unique common fixed point of f and S.

If we take $f = g$ in the theorem 3.1 we get the following interesting result.

Corollary 3.8. *Let (X, Γ) be a probabilistic metric space. Let f, S and T be self maps of X and the pairs (f, S) and (f, T) are each occasionally weakly compatible. If*

$$\int_0^{F_{fx,fy}} \phi(s)ds > \int_0^{m(x,y)} \phi(s)ds \tag{31}$$

for each $x, y \in X$, where, $m(x, y) = \min\{F_{Sx,Ty}, F_{Sx,fx}, F_{Ty,fy}, F_{Sx,fy}, F_{Ty,fx}\}$ and $\phi : [0, \infty) \to [0, \infty)$ is a Lebesgue integrable mapping which mapping on each compact subset of $[0, \infty)$ and such that for all $\epsilon > 0$,

$$\int_0^{\epsilon} \phi(s)ds > 0. \tag{32}$$

Then there is unique common fixed point of f, S and T.

Conclusion

Many of the concepts related to pair of self maps from metric space can be extended to probabilistic metric space. Further fixed point theorems that use contractive condition of integral type also can be extended to probabilistic metric space.

Acknowledgement

Authors are thankful to the learned referees for their critical remarks.

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